Moscow Institute of Physics and Technology
School of Applied Mathematics and Computer Science

First Russian–Hungarian Workshop on Discrete Mathematics

April 21 — April 23, 2017

Mini–Conference "Diophantine Problems" in occasion of Professor Nikolay Moshchevitin's 50th birthday

April 24 — April 25, 2017

Program and Abstract Book

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First Russian–Hungarian Workshop on Discrete Mathematics

Schedule

Friday, 21st of April

12:00 — 12:40	Béla Bollobás	Reconstruction
12:50 — 13:30	Maksim Zhukovskii	On the logical complexities of Subgraph Isomorphism of Subgraph Isomorphism and Induced Subgraph Isomorphism problems
Lunch		
15:00 — 15:40	János Pach	VC-dimension and the Erdos-Hajnal conjecture
15:50 — 16:30	Nikolay Dolbilin	Delaunay Sets with Point-Transitive Groups
Coffee break		
17:00 — 17:40	Attila Sali	Forbidden Pairs of Minimal Quadratic and Cubic Configurations
17:45 — 18:25	Mikhail Tikhomirov	Distance and Multidistance Graph Recognition Complexity

Saturday, 22nd of April

10:00 — 10:40	Roman Karasev	Dependence of the heavily covered point on parameters
10:50 — 11:30	Miklós Simonovits	New results in Anti-Ramsey theory
Coffee break		
12:00 — 12:40	Zoltán Füredi	Combinatorial geometry problems and Turan hypergraphs
12:45 — 13:25	Danila Cherkashin	On panchromatic colorings of uniform hypergraphs
13:30 — 13:50	Dmitry Zakharov	Acute angles
Lunch		
15:00 — 15:40	Endre Csóka	Limit theory for combinatorial problems
15:50 — 16:30	Ilya Shkredov	Any small multiplicative subgroup is not a sumset
Coffee break		
17:00 — 17:40	Dániel Gerbner	Generalized forbidden subposet problems
17:50 — 18:30	Grigory Kabatiansky	On a generalization of families of finite sets in which no set is covered by the union of two others
19:00 Conference Dinner		

Sunday, 23rd of April

9:30 — 10:10	Gyula O.H. Katona	A general 2-part Erdős-Ko-Rado theorem
10:15 — 10:55	Andrey Kupavskii	Families with forbidden subconfigurations
Coffee break		
11:20 — 12:00	Dezsõ Miklós	On the vertex sign balance of (hyper)graphs
12:05 — 12:45	Dmitry Shabanov	Estimating r-colorability threshold in a random hypergraph: a simple approach to the second moment method
12:50 — 13:30	András Recski	Interval graphs with applications
Lunch		
14:40 — 15:20	Daniel Soltész	Two extremal problems about Hamiltonian paths and cycles
15:20 — 16:00	Margarita Akhmejanova	Colorings of b-simple hypergraphs

Reconstruction

Béla Bollobás
(University of Cambridge, UK)

The *reconstruction problem* for a family $\mathcal{F}$ of finite structures asks whether it is possible to reconstruct every structure $F \in \mathcal{F}$ from the 'deck' of all its substructures of a certain kind. The best known problem of this type is the sixty-year-old Kelly—Ulam graph reconstruction conjecture stating that every graph of order $n \geq 3$ can be reconstructed from the deck consisting of its n subgraphs or order $n-1$. In my talk I shall review some of the results concerning various reconstruction problems, and sketch some of the results Paul Balister, Bhargav Narayanan and I have obtained on the problem of Mossel and Ross on the 'shotgun assembly' of labelled graphs.

On the logical complexities of Subgraph Isomorphism and Induced Subgraph Isomorphism problems

Maksim Zhukovskii
(Moscow Institute of Physics and Technology, Russia)

Consider a connected graph F with ℓ vertices. The existence of a subgraph isomorphic to F can be defined in first-order logic with quantifier depth ℓ. Note that there is no such a sentence with smaller quantifier depth, because no first-order sentence of quantifier depth less than ℓ can distinguish between the complete graphs K_ℓ and $K_{\ell-1}$. For the problem of existence of an induced subgraph isomorphic to F, the situation is not so obvious: we show that there are graphs F such that the respective property can be expressed by a sentence of quantifier depth less than the number of vertices of F.

Let C be a first-order definable class of graphs and π be a graph parameter. Let $D^k_\pi(C)$ denote the minimum quantifier depth of a first-order sentence Φ such that, for every connected graph G with $\pi(G) \geq k$, Φ is true on G exactly when G belongs to C. Note that $D^k_\pi(C) \geq D^{k+1}_\pi(C)$, and define $D_\pi(C) = \min_k D^k_\pi(C)$. In other words, $D_\pi(C)$ is the minimum quantifier depth of a first-order sentence defining C over connected graphs with sufficiently large values of π. The *variable width* of a first- order sentence Φ is the number of first-order variables used to build Φ; different occurrences of the same variable do not count. Similarly to the above, by $W_\pi(C)$ we denote the minimum variable width of Φ defining C over connected graphs with sufficiently large π. Note that $W_\pi(C) \leq D_\pi(C)$. We will consider the depth $D_\pi(C)$ and the width $W_\pi(C)$ for three parameters π, namely the number of vertices $v(G)$, the treewidth $\mathrm{tw}(G)$, and the connectivity $\kappa(G)$. Obviously, $D_v(C) \geq D_{\mathrm{tw}}(C) \geq D_\kappa(C)$ and $W_v(C) \geq W_{\mathrm{tw}}(C) \geq W_\kappa(C)$. Let $S(F)$ denote the class of graphs containing F as a subgraph, and $S[F]$ denote a class of graphs containing F as an induced subgraph.

We show that $D_v(S(F)) \leq \ell - 3$ for some F, where ℓ is the number of vertices in F. On the other hand, we show limitations of this approach by proving that $W_v(S(F)) \geq (\ell-1)/2$ for all F. The last barrier (as well as any lower bound in terms of ℓ) can be overcome by definitions over graphs with sufficiently large treewidth. Specifically, for every ℓ and $a \leq \ell$ there is an ℓ-vertex F such that $D_{\mathrm{tw}}(S(F)) \leq a$ and, moreover, $\mathrm{tw}(S(F)) = a - 1$. On the other hand, $W_{\mathrm{tw}}(S(F)) \geq \mathrm{tw}(S(F))$ for all F.

We also prove general lower bounds $D(S[F]) > e/\ell$ and $W(S[F]) > \max\{\frac{1}{2}\ell - 2\log\ell + 2, \chi\}$, where e denotes the number of edges in F and χ denotes the chromatic number of F. In fact, the first bound holds true even for $D_\kappa(S[F])$. Moreover, we find the values of all the above parameters for all connected graphs on at most 4 vertices.

VC-dimension and the Erdos-Hajnal conjecture

János Pach
(Ecole Polytechnique Fédérale de Lausanne, Switzerland)

The *Vapnik-Chervonenkis dimension* (in short, VC-dimension) of a *graph* is defined as the VC-dimension of the set system induced by the neighborhoods of its vertices. We show that every n-vertex graph with bounded VC-dimension contains a clique or an independent set of size at least $e^{(\log n)^{1-o(1)}}$. The dependence on the VC-dimension is hidden in the $o(1)$ term. This improves the general lower bound, $e^{c\sqrt{\log n}}$, due to Erdős and Hajnal, which is valid in the class of graphs satisfying any fixed nontrivial hereditary property. Our result nearly matches the celebrated Erdős-Hajnal conjecture, according to which one can always find a clique or an independent set of size at least $e^{\Omega(\log n)}$. This partially explains why most geometric intersection graphs arising in discrete and computational geometry have exceptionally favorable Ramsey-type properties. Joint work with Jacob Fox and Andrew Suk.

Delaunay Sets with Point-Transitive Groups

Nikolay Dolbilin
(Steklov Mathematical Institute, Russia)

A point set $X \subset \mathbb{R}^d$ is called a *Delauney set* if for some positive numbers r and R the two following conditions hold:
a ball $B_y(r)$ of radius r centered at any pt $y \in \mathbb{R}^d$ contains *at most one* pt $x \in X$;
a ball $B_y(R)$ of radius R centered at any pt y contains *at least one* pt $x \in X$.

Delaunay sets is a quite adequate model of the atomic structure of an arbitrary condensed matter. However, such well-organized atomic structures as crystals are described as Delaunay sets of a very special sort, namely, Delaunay sets X with a point-transitive symmetry group, i.e. with such a symmetry group that for any two points x and x' of X there is a symmetry g of X such that $g(x) = x'$. The Delaunay sets with point-transitive groups are called *regular systems*

Local theory of regular systems is aimed to rigorously derive existence of a point-transitive group for a Delaunay set X from pairwise congruence of neighborhoods of points of X. The main problem here is to estimate a radius of those neighborhoods to provide regularity of the set. Local theory is directly related to an attempt to explain why during the phase transition from liquid to solid state an atomic structure of matter moves from amorphous state into well-organized, periodic structure with a rich symmetry group.

In the talk it is supposed to discuss several key results of the local theory of regular systems.

References.

[1] Delone B.N., Dolbilin N.P., Stogrin M.I., Galiuilin R.V. (1976) A local criterion for regularity of a system of points. Soviet Math. Dokl., 17, 319—322.
[2] N. P. Dolbilin, J. C. Lagarias, M. Senechal, Multiregular point systems, Discrete Comput.

Geom., 20:4 (1998), 477—498.
[3] Dolbilin N. (2015) A Criterion for crystal and locally antipodal Delone sets. Vestnik Chelyabinskogo Gos. Universiteta, 3 (358), 6–17 (in Russian).
[4] Dolbilin N.P., Magazinov A.N. (2016) The Uniqueness Theorem for Locally Antipodal Delone Sets, "Modern Problems of Mathematics, Mechanics and Mathematical Physics", II, Collected papers, Steklov Institute Proceedings, 294, MAIK, M..
[5] Dolbilin N. (2016) Delone Sets: Local Identity and Global Symmetry, 16 pp., Springer, in print, arXiv:1608.06842.

Forbidden Pairs of Minimal Quadratic and Cubic Configurations

Attila Sali
(Rényi Institute of Mathematics, Hungary)

A matrix is *simple* if it is a (0,1)-matrix and there are no repeated columns. Given a (0,1)-matrix F, we say a matrix A has F as a *configuration*, denoted $F \prec A$, if there is a submatrix of A which is a row and column permutation of F. Let $|A|$ denote the number of columns of A. A simple (0,1)-matrix A can be considered as vertex-edge incidence matrix of a hypergraph without repeated edges. A configuration is a trace of a subhypergraph of this hypergraph. Let $\mathcal{F}$ be a family of matrices. We define the extremal function $\text{forb}(m, \mathcal{F}) = \max\{|A| \colon A$ is an $m-$ rowed simple matrix and has no configuration $F \in \mathcal{F}\}$. We consider pairs $\mathcal{F} = \{F_1, F_2\}$ such that F_1 and F_2 have no common extremal construction and derive that individually each $\text{forb}(m, F_i)$ has greater asymptotic growth than $\text{forb}(m, \mathcal{F})$, extending research started by Anstee and Koch. They determined $\text{forb}(m, \{F, G\})$ for all pairs $\{F, G\}$, where both members are *minimal quadratics*, that is both $\text{forb}(m, F) = \Theta(m^2)$ and $\text{forb}(m, G) = \Theta(m^2)$, but no proper subconfiguration of F or G is quadratic. We take this one step further. That is, we consider cases when one of F or G is a simple minimal cubic configuration and the other one is a minimal quadratic or minimal simple cubic. We solve all cases when the minimal simple cubic configuration has four rows. If a conjecture of Anstee is true, then there is no minimal simple cubic configuration on 5 rows. About the six-rowed ones we observe that $\text{forb}(m, \{F, G\})$ is quadratic if F is minimal quadratic and G is a 6-rowed minimal cubic in all, but one cases. In the remaining case we believe that non-existence of common quadratic product construction indicates that the order of magnitude is $o(m^2)$.

On the logical complexities of Subgraph Isomorphism and Induced Subgraph Isomorphism problems

Mikhail Tikhomirov
(Moscow Institute of Physics and Technology, Russia)

We discuss complexity of realizability of certain kinds of geometric graphs induced by distances among a set of points in $\mathbb{R}^d$. The first and foremost of these are the celebrated *unit-distance graphs*, that is, induced subgraphs of $\Gamma(\mathbb{R}^2)$ — the graph with vertices in every point of the plane, and edges connecting each pair at Euclidean distance 1. Numerous discrete geometry open questions are concerned with bounding key characteristics of unit-distance graphs, such as the number of edges as a function of the number of vertices n (the question is known as the *unit-distance problem*) or chromatic number ([finitary version of] Hadwiger-Nelson problem). It is natural to extend the notion (and corresponding problems) to a space of arbitrary dimension $\mathbb{R}^d$.

A natural question is: how fast a given graph can be classified as (isomorphic to) a unit-distance graph in a given dimension? The question is trivial for $d = 1$, but becomes hard very quickly. Horvat et al. shown NP-hardness of unit-distance graph recognition for $d = 2$, and Schaefer established that the $d = 2$ case of the problem is complete for the existential theory of the reals (which is a considerable evidence against NP membership). NP-hardness of the $d > 2$ case was declared to be proved by Horvat et al. in the aforementioned paper, but the proof was refuted by Raigodoskii, and the result was recently reestblished via a different method by the present author. The latter proof also works for slightly different classes of graphs, such as non-induced subgraphs of $\Gamma(\mathbb{R}^d)$, or graphs admitting homomorphism/subgraph isomorphism to $\Gamma(\mathbb{R}^d)$.

We also consider a multidistance case of the same problem. For a set of positive numbers $\mathcal{A}$ let us construct $\Gamma_{\mathcal{A}}(\mathbb{R}^d)$ as the graph with vertices in $\mathbb{R}^d$, and edges between all pairs such that their Euclidean distance is an element of $\mathcal{A}$. We call an induced subgraph of $\Gamma_{\mathcal{A}}(\mathbb{R}^d)$ an *$\mathcal{A}$-distance graph in* $\mathbb{R}^d$. Complexity of recognizing $\mathcal{A}$-distance graphs and their variations is largely unstudied. As an initial effort in studying this, it was recently shown by the author that when $\mathcal{A}$ is finite, recognition of $\mathcal{A}$-distance graphs in $\mathbb{R}^1$ is in NP if $\mathcal{A}$ is a set of integers up to a common multiplier, and NP- complete otherwise. Note that possible connected components of $\Gamma_{\mathcal{A}}(\mathbb{R}^d)$ with finite $\mathcal{A}$ are exactly the Cayley graphs of free abelian finitely generated groups, hence the result has an additional algebraic motivation.

There are interesting open questions in the field. It feels important to prove $\exists\mathbb{R}$-hardness of unit-distance graph recognition in higher dimensions, or, for that matter, at least present any $\exists\mathbb{R}$-hard geometric problem that is more-than-two-dimensional in nature. Proving hardness of $\mathcal{A}$-distance realizability in the plane or higher dimensions is an interesting challenge in robust gadget construction. Also, allowing $\mathcal{A}$ to be a union of intervals, it is possible to obtain "real-life" imprecise versions of the aforementioned problems, along with very practical inquiries about their complexity.

Dependence of the heavily covered point on parameters

Roman Karasev
(Moscow Institute of Physics and Technology, Russia)

The "first selection lemma" by Imre Bárány asserts that given many points in $\mathbb{R}^n$ it is possible to find another point p such that the probability that it is covered by a randomly chosen simplex formed by some $n + 1$ of the given points is at least some positive constant c_n.

Recently Gromov designed a new technique to prove such results, improving the constant c_n. We examine GromovвЂ™s method in order to understand the dependence of such a "heavily covered" point on parameters. We cannot prove the continuous dependence on parameters, but manage to utilize the "homological continuous dependence" of the heavily covered point. This allows us to infer some corollaries in a usual way.

We also give an elementary argument to prove the simplest of these corollaries: Given several straight lines in the plane in general position, it is possible to find a point that is surrounded by a randomly chosen triple of lines with probability at least 2/9.

This is a joint work with Alexey Balitskiy and Uli Wagner.

New results in Anti-Ramsey theory

Miklós Simonovits
(Rényi Institute of Mathematics, Hungary)

Whenever in an extremal graph problem the extremal structure has a simple structure and the almost-extremal structures have structures very similar to the extremal ones, we may try to use the Stability method to get sharp results for large cases.

This can be applied very successfully in extremal graph theory, in Anti-Ramsey problems and also in "dual" Anti-Ramsey problems.

Burr, Erdős, Graham and T. Sós defined and investigated a *dual* variant of the Anti-ramsey problems. (The problem, being interesting on its own, came from some applications in Theoretical Computer Science.) As they pointed out, one of the most interesting cases they could not settle was that of C_5.

Here we improve several of their results. We shall prove, among others, that if a graph G_n has $e = n + 1$ edges and we colour its edges so that every $C_5 \subset G_n$ is 5–coloured, then we have to use at least $n + 3$ colours, if n is sufficiently large. This result is sharp.

Some results in this lecture are joint with Paul Erdős, from the late 1980's.

Combinatorial geometry problems and Turan hypergraphs

Zoltán Füredi
(Rényi Institute of Mathematics, Hungary,
University of Illinois at Urbana—Champaign, USA)

We overlook a few applications of using extremal hypergraphs in combinatorial geometry questions.

A sample result: Let $h(n)$ be the maximum number of triangles among n points on the plane which are almost regular (all three angles are between 59 to 61 degrees). Conway, Croft, Erdos and Guy (1979) proved an upper bound for $h(n)$ and conjectured that $h(n) = (1 + o(1))n^3/24$. We prove this (and other) conjectures. Among our main tools we use Razborov's flag algebra method to determine the Turan numbers of certain 3-uniform hypergraphs. Several problems remain open. This is a joint work with Imre Barany (with some computer help from Manfred Scheucher).

On panchromatic colorings of uniform hypergraphs

Danila Cherkashin
(Saint Petersburg State University, Russia)

A hypergraph is a pair (V, E), where V is a finite set whose elements are called vertices and E is a family of subsets of V, called edges. A hypergraph is n-uniform if every edge has size n. A vertex r-coloring of a hypergraph (V, E) is a map $c : V \to \{1, \ldots, r\}$.

An r-coloring of vertices of a hypergraph is called *panchromatic* if every edge contains a vertex of every color. The problem of the existence of a panchromatic coloring of a hypergraph was stated in the local form by P. Erdős and L. Lovász in [5]. They proved that if every edge of an n-uniform hyperhraph intersects at most $r^{n-1}/4(r-1)^n$ other edges then the hypergraph has a panchromatic r-coloring.

We study the quantity $p(n, r)$, that is the minimal number of edges of an n-uniform hypergraph without panchromatic coloring in r colors.

Upper bounds

Using the results from [7] A. Kostochka proved [1] that for some constants c_1, $c_2 > 0$

$$\frac{1}{r} e^{c_1 \frac{n}{r}} \leq p(n, r) \leq r e^{c_2 \frac{n}{r}}. \tag{1}$$

In works [2, 4] D. Shabanov gives the following upper bounds:

$$p(n, r) \leq c \frac{n^2 \ln r}{r^2} \left(\frac{r}{r-1} \right)^n, \text{ if } 3 \leq r = o(\sqrt{n}), n > n_0;$$

$$p(n, r) \leq c \frac{n^{3/2} \ln r}{r} \left(\frac{r}{r-1} \right)^n, \text{ if } r = O(n^{2/3}) \text{ and } n_0 < n = O(r^2);$$

$$p(n, r) \leq c \max \left(\frac{n^2}{r}, n^{3/2} \right) \ln r \left(\frac{r}{r-1} \right)^n \text{ for all } n, r \geq 2.$$

The following theorem gives better upper bound in the case when $n = o(r^{3/2})$.

Theorem 1. *The following inequality holds for every $n \geq 2$, $r \geq 2$*

$$p(n, r) \leq c \frac{n^2 \ln r}{r} \left(\frac{r}{r-1} \right)^n.$$

Lower bounds

We start by noting that an evident probabilistic argument gives $p(n, r) \geq \frac{1}{r} (\frac{r}{r-1})^n$. This gives lower bound (1) with $c_1 = 1$. This was essentially improved by D. Shabanov in [2]:

$$p(n, r) \geq c \frac{1}{r^2} \left(\frac{n}{\ln n} \right)^{1/3} \left(\frac{r}{r-1} \right)^n \text{ for } n, r \geq 2, r < n.$$

Next, A. Rozovskaya and D. Shabanov [6] showed that

$$p(n, r) \geq c \frac{1}{r^2} \sqrt{\frac{n}{\ln n}} \left(\frac{r}{r-1} \right)^n \text{ for } n, r \geq 2, r \leq \frac{n}{2 \ln n}.$$

Using the Alterations method (see Section 3 of [3]) we can get the following lower bound for all the range of n, r. It gives better results when $r \geq c\sqrt{n}$. Note that it is the first lower bound in the case when n/r is sufficiently small.

Theorem 2. *For $n \geq r \geq 2$ holds*

$$p(n,r) \geq c\frac{r}{n}e^{\frac{n}{r}}.$$

Also we produce another way to get better bound in the case $n \geq cr^2$, which admits a local version.

References

[1] KOSTOCHKA, ALEXANDR *On a theorem of Erdős, Rubin, and Taylor on choosability of complete bipartite graphs*, The Electronic Journal of Combinatorics **9** (9) (2002).

[2] SHABANOV, DMITRII A. *The existence of panchromatic colourings for uniform hypergraphs.* Sbornik: Mathematics **201** (4) (2010) 607.

[3] ALON, NOGA AND SPENCER, JOEL H. AND ERDŐS, PAUL *The probabilistic method.* John Wiley & Sons (2016).

[4] SHABANOV, DMITRY A. *On a generalization of Rubin's theorem.* Journal of Graph Theory **67** (3) (2011) 226–234.

[5] ERDŐS, PAUL AND LOVÁSZ, LÁSZLÓ *Problems and results on 3-chromatic hypergraphs and some related questions.* Infinite and finite sets **10** (2) (1975) 609–627.

[6] ROZOVSKAYA, ANASTASIYA P. AND SHABANOV, DMITRII A. *Improvement of the lower bound in the Kostochka problem of panchromatic coloring of a hypergraph.* Mathematical Notes **89** (5) (2011) 903–906.

[7] ALON, NOGA *Choice numbers of graphs: a probabilistic approach.* Combinatorics, Probability and Computing **1** (2) (1992) 107–114.

Acute angles

Dmitry Zakharov
(School 179, Russia)

Let $f(n)$ be the maximum number of points in $\mathbb{R}^n$ that generate only acute-angled triangles. Danzer and Grünbaum conjectured that $f(n) = 2n + 1$. Erdős and Füredi disproved this conjecture and showed using probability that $f(n) \geq (1.154 + o(1))^n$. This bound has been successively improved, so that the current record is $(1.201 + o(1))^n$ and it is also based on a probabilistic argument. In our talk, we give an explicit construction showing that $f(n) \geq (1.26 + o(1))^n$.

Limit theory for combinatorial problems

Endre Csóka
(Rényi Institute of Mathematics, Hungary)

We present limit approach to three closely related problems: Alpern's Caching Game, Manickam-MiklΓis-Singhi Conjecture and Kikuta-Ruckle Conjecture. We show a very counter-intuitive result about Alpern's Caching Game, and we extend conjectures by finding the structure in the seemingly chaotic properties of the solutions. But the main focus of the talk is on the limit approach, which combines asymptotic analysis and the use of limit problems.

Any small multiplicative subgroup is not a sumset

Ilya Shkredov
(Steklov Mathematical Institute, Russia)

We prove that for an arbitrary $\varepsilon > 0$ and any multiplicative subgroup $\Gamma \subseteq \mathbf{F}_p$, $1 \ll |\Gamma| \le p^{2/3-\varepsilon}$ there are no sets B, $C \subseteq \mathbf{F}_p$ with $|B|, |C| > 1$ such that $\Gamma = B + C$. Also, we obtain that for $1 \ll |\Gamma| \le p^{6/7-\varepsilon}$ and any $\xi \ne 0$ there is no a set B such that $\xi\Gamma + 1 = B/B$.

Generalized forbidden subposet problems

Dániel Gerbner
(Rényi Institute of Mathematics, Hungary)

A subfamily $\{F_1, F_2, \ldots, F_{|P|}\} \subseteq \mathcal{F}$ of sets is a copy of a poset P in $\mathcal{F}$ if there exists a bijection $\phi : P \to \{F_1, F_2, \ldots, F_{|P|}\}$ such that whenever $x \le_P x'$ holds, then so does $\phi(x) \subseteq \phi(x')$. For a family $\mathcal{F}$ of sets, let $c(P, \mathcal{F})$ denote the number of copies of P in $\mathcal{F}$, and we say that $\mathcal{F}$ is P-free if $c(P, \mathcal{F}) = 0$ holds. For any two posets P, Q let us denote by $La(n, P, Q)$ the maximum number of copies of Q over all P-free families $\mathcal{F} \subseteq 2^{[n]}$, i.e. $\max\{c(Q, \mathcal{F}) : \mathcal{F} \subseteq 2^{[n]}, c(P, \mathcal{F}) = 0\}$. This generalizes the well-studied parameter $La(n, P) = La(n, P, P_1)$ where P_1 is the one element poset. We consider the problem of determining $La(n, P, Q)$ when P and Q are small posets, like chains, forks, the N poset, etc and when Q is a complete multi-level poset.

Joint work with Balazs Keszegh and Balazs Patkos.

On a generalization of families of finite sets in which no set is covered by the union of two others

Grigory Kabatiansky
(Skolkovo Institute of Science and Technology (Skoltech), Russia)

A family $\mathbf{F}$ of k-subsets of a n-set X called t-IPP family of sets if for any k-subset which belongs to the union of some t sets of $\mathbf{F}$ at least on of these sets can be uniquely determine. This notion was introduced as a combinatorial model for traitor tracing in broadcast encryption by Chor, Fiat and Naor [1] for rather general scenario, and this particular form was given in [2].

It is clear that a t-IPP family of sets is a family of finite sets in which no set is covered by the union of t others - the notion developed in [3], [4], also known in coding theory as superimposed codes [5], [6]. Some technique of [3], [4] was used in [7], [8] in order to obtain upper bounds on the cardinality of t-IPPS families. A bit surprising but no good lower asymptotic bound was known.

In this talk we derive an asymptotic bound only for the case $t = 2$. Despite some analogies between the notion of t-IPP family of sets and the notion of t-IPP codes, see [9], we failed to extend the technique of [10] to the case of arbitrary t. Moreover known upper and lower bounds for the rate of the corresponding codes differs in ten times, what sounds like we should improve bounds - lower, upper or both.

The talk is based on a joint paper with Elena Egorova.

References

[1] B. Chor, A. Fiat, M. Naor. Tracing traitors. Crypto-93, Springer , 257-270, 1994.

[2] D. R. Stinson, R. Wei. Combinatorial properties and constructions of traceability schemes and frameproof codes, SIAM Journal on Discrete Mathematics, 11(1), 41-53, 1998.

[3] P. Erdos, P. Frankl, Z. Furedi. Families of finite sets in which no set is covered by the union of two others, Journal of Combinatorial Theory, Series A, Vol.33, N 2, pp. 158-166, 1982.

[4] Z. Furedi, P. Erdos, P. Frankl. Families of finite sets in which no set is covered by the union of r others, Israel J. Math. 51(1), 79-89, 1985.

[5] W. Kautz, R. Singleton. Nonrandom binary superimposed codes, IEEE Transactions on Information Theory 10(4), 363-377, 1964.

[6] A.G. Dyachkov, V.V. Rykov. Bounds on the Length of Disjunctive Codes, Problems of Information Transmission, v.18, N2, 166-171, 1982.

[7] M. J. Collins. Upper bounds for parent-identifying set systems. Designs, Codes and Cryptography, 51(2), 167-173, 2009.

[8] Y. Gu, Y. Miao. Bounds on Traceability Schemes. arXiv preprint arXiv:1609.08336, 2016.

[9] H. D. Hollmann, J.H. van Lint, J. P. Linnartz, L. M. Tolhuizen. On codes with the identifiable parent property. Journal of Combinatorial Theory, Series A, 82(2), 121-133, 1998.

[10] A. Barg, G. Cohen, S. Encheva, G. Kabatiansky, G. Zémor. A hypergraph approach to the identifying parent property: the case of multiple parents. SIAM Journal on Discrete Mathematics, 14(3), 423-431, 2001.

A general 2-part Erdős-Ko-Rado theorem

Gyula O.H. Katona
(Rényi Institute of Mathematics, Hungary)

A two-part extension of the famous Erdős-Ko-Rado Theorem is proved. The underlying set is partitioned into X_1 and X_2. Some positive integers $k_i, \ell_i (1 \leq i \leq m)$ are given. We prove that if $\mathcal{F}$ is an intersecting family containing members F such that $|F \cap X_1| = k_i, |F \cap X_2| = \ell_i$ holds for one of the values $i (1 \leq i \leq m)$ then $|\mathcal{F}|$ cannot exceed the size of the largest subfamily containing one element. The statement was known for the case $m = 2$ as a results of Frankl.

Families with forbidden subconfigurations

Andrey Kupavskii
(Moscow Institute of Physics and Technology, Russia,
Ecole Polytechnique Fédérale de Lausanne, Switzerland)

Put $[n] := \{1, 2, \ldots, n\}$ and let $2^{[n]}$ denote the power set of $[n]$. A subset $\mathcal{F} \subset 2^{[n]}$ is called a *family of subsets of* $[n]$, or simply a *family.*

The maximum number of pairwise disjoint members of a family $\mathcal{F}$ is denoted by $\nu(\mathcal{F})$ and is called the *matching number* of $\mathcal{F}$. It is a classical question due to Erdős to determine, how big a family $\mathcal{F} \subset 2^{[n]}$ could be, if $\nu(\mathcal{F}) < s$ for some integer s. Let us denote this number $e(n, s)$. The value $e(n, s)$ was found by Kleitman [4] for $n = sm, sm - 1$. Quinn [5] found the value of $e(3m + 1, 3)$.

In our recent work, we determined the values $e(sm - 2, s)$ for all m, as well as the values of $e(sm - l, s)$ for $s > lm + 3l + 3$. In this talk, I wanted to discuss the method we used to reprove Quinn's result, as well as to determine $e(4m + 2, 4)$ (see [1]). This method seems to be rather general, and it already allowed us to resolve completely the following problem, also posed by Erdős: what is the maximum size of $\mathcal{F} \subset 2^{[n]}$, such that $\mathcal{F}$ does not contain two disjoint sets and their union? The answer for $n = 3m + 1$ was obtained by Kleitman [3], and we settled [2] the other two cases: $n = 3m$ and $n = 3m + 2$.

Joint work with Peter Frankl. Research supported by the grant RNF 16-11-10014.

References

[1] P. Frankl, A. Kupavskii, *The largest families of sets with no matching of sizes 3 and 4*, arXiv:1701.04107

[2] P. Frankl, A. Kupavskii, *Families of sets with no two disjoint sets and their union*, preprint

[3] D.J. Kleitman, *On families of subsets of a finite set containing no two disjoint sets and their union*, J. Combin. Theory 5 (1968), N3, 235–237.

[4] D.J. Kleitman, *Maximal number of subsets of a finite set no k of which are pairwise disjoint*, Journ. of Comb. Theory 5 (1968), 157–163.

[5] F. Quinn, PhD Thesis, Massachusetts Institute of Technology (1986).

On the vertex sign balance of (hyper)graphs

Dezsõ Miklós
(Rényi Institute of Mathematics, Hungary)

Pokrovskiy and, independently, Alon, Huang and Sudakov introduced the MMS (Manickam-Miklos-Singhi) property of hypegraphs: "for every assignment of weights to its vertices with non-negative overall sum, the number of edges whose total weight is non-negative is at least the minimum degree of H". This immediately leads to the definition of the following hypergraph parameter: The vertex sign balance of a hypergraph is the minimum number edges whose total weight is non-negative, where the minimum is taken over all assignments of weights to the vertices with non-negative overall sum. The vertex sign balance is always between 0 and the minimum degree of the (hypergraph), both bound being sharp. General and special properties (for graphs or three uniform hypergraphs) of this parameter will be presented. In particular, the characterization of the vertex sign balance of the graphs leads to the result that the question if a (hyper)graph has the MMS property is NP-complete.

Estimating r-colorability threshold in a random hypergraph: a simple approach to the second moment method

Dmitry Shabanov
(Moscow Institute of Physics and Technology,
Moscow State University, Russia)

The talk deals with estimating the probability threshold for r-colorability of a random hypergraph. Let $H(n,k,p)$ denote the classical binomial model of a random k-uniform hypergraph: every edge of a complete k-uniform hypergraph on n vertices is included into $H(n,k,p)$ independently with probability $p \in (0,1)$.

We study the question of estimating the probability threshold for the r-colorability property of $H(n,k,p)$. It is well known that for fixed $r \geqslant 2$ and $k \geqslant 2$, this threshold appears in a sparse case when the expected number of edges is a linear function of n: $p\binom{n}{k} = cn$ for some fixed $c > 0$.

The following result gives a new lower bound for the r-colorability threshold.

Theorem 3. *Let $k \geqslant 4$, $r \geqslant 2$ be integers and $c > 0$. Then there exist absolute constants $C > 0$ and $d_0 > 0$ such that if $\max(r,k) > d_0$ and*

$$c < r^{k-1}\ln r - \frac{\ln r}{2} - \frac{r-1}{r} - C\cdot\frac{k^2 \ln r}{r^{k/3-1}}, \tag{2}$$

then

$$\mathsf{Pr}\left(H(n,k,cn/\binom{n}{k}) \text{ is } r\text{-colorable}\right) \to 1 \text{ as } n \to +\infty.$$

Theorem 3 improves the previous result from [1] and provides a bounded gap with the known upper bound. The estimate (2) is only $\frac{r-1}{r} + O\left(\cdot\frac{k^3\ln r}{r^{k/3-1}}\right)$ less than the upper bound from [1]. In the case of two colors, $r = 2$, the result coincides with the estimate obtained in [2].

The proof of Theorem 3 is based on the new approach to the second moment method. We also provide some extensions of the used technique.

The work is supported by the Russian Science Foundation under grant N 16-11-10014.

References

[1] M. Dyer, A. Frieze, C. Greenhill, “On the chromatic number of a random hypergraph”, *Journal of Combinatorial Theory, Series B*, **113** (2015), 68–122.

[2] D. Achlioptas, C. Moore, “Random k-SAT: two moments suffice to cross a sharp threshold”, *SIAM Journal on Computing*, **36**:3 (2005), 740–762.

Interval graphs with applications

András Recski
(Technical University, Budapest, Hungary)

Interval graphs form a subclass of perfect graphs, which can be represented by a collection of intervals (vertices correspond to the intervals and edges to nonempty intersections of the corresponding intervals). We survey some classical and some more recent results and present some applications, mostly in the design of very large scale (VLSI) integrated circuits.

Two extremal problems about Hamiltonian paths and cycles

Dániel Soltész
(Rényi Institute of Mathematics, Hungary)

The first part is joint work with Ron Aharoni. The first problem in its original form is the following: How many vertex disjoint Schrijver subgraphs are there in the Kneser graph? We can reformulate this problem naturally in the language of Hamiltonian cycles: By the union of two graphs on the same vertex set we mean the union of the edge sets. What is the maximal number of Hamiltonian cycles on n vertices such that the independence number of every pairwise union is at most cn for a fixed constant c. We show that there is a threshold with the property that for every c under the threshold the answer is at most a constant, independent from n. but for every c above the constant the answer is exponential in n.

The second part is joint work with István Kovács. The second problem is motivated by permutations. Several questions of the following type are studied. Suppose that we are given a compatibility relation between pairs of permutations. What is the maximal number of pairwise compatible permutations? Many such problems (that are ultimately motivated by coding theory) can be formulated in terms of Hamiltonian paths of the complete graph K_n. In these cases the compatibility relation can be often described as "Do the two Hamiltonian paths contain a specific subgraph in heir union?". We solve this problem where the specific graph is a triangle. This is part of an ongoing investigation where we aim to get a better intuition that for a given compatibility relation should one expect an answer of exponential size or larger.

Colorings of b-simple hypergraphs

Margarita Akhmejanova
(Moscow State University, Russia)

This work deals with the problem concerning estimating the maximum edge degree in n-uniform b-simple hypegraphs with high chromatic number.

Recall some basic definitions. Hypergraph is a pair (V, E) where V is a set, called the *vertex set* of the hypergraph and E is a family of subsets of V, whose elements are called the *edges* of the hypergraph. A hypergraph is *n-uniform* if every of its edges contains exactly n vertices. The *degree of an edge A* in a hypergraph H is the number of other edges of H which have nonempty intersection with A. The maximum edge degree of H is denoted by $\Delta(H)$.

An r-coloring of hypergraph $H = (V, E)$ is a mapping from the vertex set V to the set of r colors, $\{0, \dots, r-1\}$. A coloring of H is called *proper* if it does not create monochromatic edges (i.e. every edge contains at least two vertices which receives different colors). A hypergraph is said to be *r-colorable* if there exists a proper r-coloring of that hypergraph.

Consider the family of b-simple hypergraphs, in which any two edges do not share more than b common vertices. The best known result is due to Kozik [1] who showed that for any b-simple n-uniform hypergraph H, the condition

$$\Delta(H) \le c(b, r)\frac{n}{\ln n} r^n \tag{3}$$

implies the r-colorability of H, where $c(r, b) > 0$ is some positive function of r and b.

The main result of the talk refines the estimate (3) as follows.

Theorem 4. *Suppose $b \ge 1$, $r \ge 2$ and $n > n_0(b)$ is large enough in comparison with b. Then if a b-simple n-uniform hypergraph H satisfies the inequality*

$$\Delta(H) \le c \cdot n\, r^{n-b}, \tag{4}$$

where $c > 0$ is some absolute constant, then H is r-colorable.

In the case of simple hypergraphs, i.e. for $b = 1$, the above result (4) is not new. It was obtained previously by Kozik and Shabanov [2]. For fixed r, b, the bound (4) is $\Theta_{r,b}(n)$ times smaller than the known upper bound proved by Kostochka and Rödl [3].

References

[1] J. Kozik, "Multipass greedy coloring of simple uniform hypergraphs", *Random Structures and Algorithms*, **48**:1 (2016), 125–146.

[2] J. Kozik, D.A. Shabanov, "Improved algorithms for colorings of simple hypergraphs and applications", *Journal of Combinatorial Theory, Series B*, **116** (2016), 312–332.

[3] A.V. Kostochka, V. Rödl, "Constructions od sparse uniform hypergraphs with high chromatic number", *Random Structures and Algorithms*, **36**:1 (2010), 46–56.

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Mini–Conference "Diophantine Problems" in occasion of Professor Nikolay Moshchevitin's 50th birthday

Schedule

Monday, 24th of April

10:30 — 11:10	Iskander Aliev	Sparse Solutions of Linear Diophantine Equations
11:25 — 12:05	Nicolas Chevallier	Almost all sequences of best simultaneous Diophantine approximations are both regular and singular
12:20 — 12:40	Natalia Dyakova	On badly approximable linear forms and isotropically winning property
Lunch		
15:00 — 15:40	Jinpeng An	Values of generalized binary quadratic forms at integral points
15:55 — 16:35	Erez Nesharim	Diophantine approximation in positive characteristic and combinatorics of infinite matrices
16:50 — 17:30	Sergei Konyagin	A construction of Schinzel–many numbers in a short interval without small prime factors
17:45 — 18:15	Alexander Prikhodko	Diophantine problems around spectral invariants of dynamical systems, and self-similar quantum solitons

Tuesday, 25th of April

10:30 — 11:10	Attila Bérczes	Effective results for Diophantine equations over finitely generated domains
11:25 — 12:05	Giedrius Alkauskas	The Minkowski question mark function, quasi-modular forms, and the Dedekind eta function
12:20 — 12:50	Dmitriy Gayfulin	Attainable numbers and the Lagrange spectrum
Lunch		
15:00 — 15:40	Yuri Nesterenko	Quasi-modular functions and transcendental numbers
15:55 — 16:35	Artūras Dubickas	On rational approximations to two irrational numbers
16:50 — 17:30	Oleg German	Klein polyhedra as multidimensional analogue of continued fractions

Sparse Solutions of Linear Diophantine Equations

Iskander Aliev
(Cardiff University, UK)

We present structural results on solutions to the underdetermined Diophantine system

$$Ax = b, x \in \mathbb{Z}^n_{\geq 0}$$

with the smallest number of nonzero entries. The proofs are based on the geometry of numbers. These results have some interesting consequences in discrete optimisation.

This is a joint work with J. De Loera, T. Oertel and C. O'Neill.

Almost all sequences of best simultaneous Diophantine approximations are both regular and singular

Nicolas Chevallier
(Université de Haute Alsace, France)

Singular vectors were introduced by Khintchine in the twenties. They can be characterized with best Diophantine approximations. Let $((q_n, p_n))_n$ be the sequence of best approximation vectors of a vector x in $\mathbb{R}^d$ with respect to the Euclidean norm. The vector x is singular if and only if the sequence $q_{n+1}\|q_n x - p_n\|^d$ goes to zero when n goes to infinity. Such vectors do not exist in the one dimensional case because of the inequality $q_{n+1}\|q_n x - p_n\| \geq 1/2$. We will study the existence of such lower bounds in higher dimensions.

On badly approximable linear forms and isotropically winning property

Natalia Dyakova
(Moscow State University, Russia)

Let Θ be an $n \times m$ matrix and let $L_1, \ldots, L_n$ be the linear forms with coefficients written in the rows of Θ. Suppose Θ is badly approximable with weights $\mathbf{k} = (k_1, \ldots, k_n)$, $k_1 + \ldots + k_n = 1$, i.e.

$$\inf_{\mathbf{q} \in \mathbb{Z}^m \setminus \{0\}} \max_{1 \leq i \leq n} (|\mathbf{q}|^{mk_i} ||L_i(\mathbf{q})||) > 0,$$

where $\| \cdot \|$ denotes the distance to the nearest integer. Recently Moshchevitin and Harrap proved that the set

$$B_\Theta(\mathbf{k}, n, m) = \left\{ \eta \in [0,1]^n \,\middle|\, \inf_{\mathbf{q} \in \mathbb{Z}^m \setminus \{0\}} \max_{1 \leq i \leq n} (|\mathbf{q}|^{mk_i} ||L_i(\mathbf{q}) - \eta_i||) > 0 \right\}$$

is winning. We improve their result by showing that this set is isotropically winning.

Values of generalized binary quadratic forms at integral points

Jinpeng An
(Peking University, China)

Kleinbock and Weiss proved that the set of indefinite binary quadratic forms Q such that the closure of $Q(Z^2)$ is disjoint from a fixed countable set has full Hausdorff dimension. In this talk, we will discuss extensions of this result to certain generalized binary quadratic forms. Proofs will be based on investigations of nondense orbits of nonquasiunipotent homogeneous flows. This is an ongoing joint work with Lifan Guan and Dmitry Kleinbock.

Diophantine approximation in positive characteristic and combinatorics of infinite matrices

Erez Nesharim
(University of York, UK)

The positive characteristic analogue of Diophantine approximation is getting much attention recently. It deals with approximations by rational functions in the field $\mathbb{F}_q\left(\left(\frac{1}{t}\right)\right)$. The study of these approximations turn out to be equivalent to the study of the rank of finite submatrices of infinite matrices over finite fields.

Many problems in Diophantine approximation are concerned with finding the numerical value of Diophantine constants. Since the field $\mathbb{F}_q\left(\left(\frac{1}{t}\right)\right)$ is equipped with a natural absolute value that assumes only countably many values, the calculation of analogous Diophantine constants is more approachable.

In this talk I will survey some Diophantine constants and their positive characteristic analogues.

A construction of Schinzel–many numbers in a short interval without small prime factors

Sergei Konyagin
(Steklov Mathematical Institute, Russia)

Assuming the prime k-tuple conjecture we prove that for a sufficiently large x there is $y \geq x$ such that $\pi(x+y) - \pi(x) - \pi(y) \gg x(\log x)^{-2} \log\log\log x$.

Diophantine problems around spectral invariants of dynamical systems, and self-similar quantum solitons

Alexander Prikhodko
(Moscow Institute of Physics and Technology, Russia)

The starting point of our investigation is the well-known construction of Riesz product

$$\sigma = \prod_{n=1}^{\infty} |P_n(t)|^2,$$

where

$$P_n(t) = e^{i\omega_1 t} + .. + e^{i\omega_n t}, \qquad t \in \mathbb{R}, \quad n \geq 2, \tag{$*$}$$

are exponential sums with coefficients zero and one, since is represents spectral measures σ for a class of zero entropy dynamical systems [2,7]. It is shown in [7] that under certain resonance conditions the following paradoxial behavior is observed.

Theorem 1. For any compact set $K \in \mathbb{R}^*$ and $\epsilon > 0$ there exists an infinite sequence of exponential sums $(*)$ satisfying estimate $\|n^{-1}|P_n|^2 - 1\|_{L^1(K,\mu^*)} < \epsilon$.

The main ingredient of the construction leading to flat exponential sums is a class of self-similar quantum dynamical systems demonstrating resonance behavior (see [1,4]), and the best way to see the effect is to use the language of Diophantine approximations. We consider an equation $i\dot{\psi} = \hat{H}\psi$ possessing a discrete component Λ in spectrum, satisfying multiplicative self-similarity $\Lambda = q^2\Lambda$, where q is a real-valued parameter close to 1 (see [4]). The problem of finding resonance states leading to flat exponential sums is stated in terms of joint Diophantine approximations of numbers $(s \ln s)_{s\in[s_0,s_1]}$ on torus of high dimension, which is related, by Khintchine's transference principle (see [6]), to estimates of linear forms

$$L(k) = k_1 \ln a_1 + \cdots + k_m \log a_m.$$

An estimate from below for $L(k)$ is given by classical Baker's theorem improved by Matveev [5]. In our special case $\log a_j = \log j$ an additional reinforcement is achieved by applying work by Nikishin [3]. At the same time, the question on finding precise estimate for return times in the original dynamical system still remains open.

References

[1] V. G. Drinfeld. Quantum groups. Differential geometry, Lie groups and mechanics. Part VIII, Zap. Nauchn. Sem. LOMI, 155, Nauka, Leningrad, 1986, 18—49; J. Soviet Math., 41:2 (1988), 898—915.

[2] M. Lemanczyk. Spectral Theory of Dynamical Systems, Encyclopedia of Complexity and System Science. Springer Verlag, 2009.

[3] E. M. Nikishin. On logarithms of natural numbers. Math. USSR-Izv., **15**:3 (1980), 523–530.

[4] V. P. Spiridonov. Exactly solvable potentials and quantum algebras. *Phys. Rev. Lett.* **69** (1992), 398–401.

[5] E. M. Matveev. An explicit lower bound for a homogeneous rational linear form in logarithms of algebraic numbers. Izv. Math., **62**:4 (1998), 723–772.

[6] O. N. German. On Diophantine exponents and Khintchine's transference principle. Mosc. J. Comb. Number Theory **2** (2012), 22–51.

[7] A. A. Prikhod'ko. Littlewood polynomials and applications of them in the spectral theory of dynamical systems. Sb. Math., **204**:6 (2013), 910–935.

Effective results for Diophantine equations over finitely generated domains

Attila Bérczes
(University of Debrecen, Hungary)

Let $A := \mathbb{Z}[z_1, \ldots, z_r] \supset \mathbb{Z}$ be a finitely generated integral domain over $\mathbb{Z}$ and denote by K the quotient field of A. Finiteness results for several kinds of Diophantine equations over A date back to the middle of the last century. S. Lang generalized several earlier results on Diophantine equations over the integers to results over A, including results concerning unit equations, Thue-equations and integral points on curves. However, all his results were ineffective.

The first effective results for Diophantine equations over finitely generated domains were published in the 1980's, when Győry developed his new effective specialization method. This enabled him to prove effective results over finitely generated domains of a special type.

In 2011 Evertse and Győry refined the method of Győry such that they were able to prove effective results for unit equations $ax + by = 1$ in $x, y \in A^*$ over arbitrary finitely generated domains A of characteristic 0. Later Bérczes, Evertse and Győry obtained effective results for Thue equations, hyper- and superelliptic equations and for the Schinzel-Tijdeman equation over arbitrary finitely generated domains.

In this talk I will present my effective results for equations $F(x, y) = 0$ in $x, y \in A^*$ for arbitrary finitely generated domains A, and for $F(x, y) = 0$ in $x, y \in \overline{\Gamma}$, where $F(X, Y)$ is a bivariate polynomial over A and $\overline{\Gamma}$ is the division group of a finitely generated subgroup Γ of K^*. These are the first effective versions of the famous corresponding ineffective results of Lang (1960) and Liardet (1974).

The Minkowski question mark function, quasi-modular forms, and the Dedekind eta function

Giedrius Alkauskas
(Vilnius University, Lithuania)

The Minkowski question mark function is a rich object which can be explored from the perspective of dynamical systems, complex dynamics, metric number theory, multifractal analysis, transfer operators, integral transforms, and as a function itself via analysis of continued fractions and convergents. Our permanent target, however, was to get an arithmetic interpretation of the moments of $?(x)$ (which are relatives of periods of Maass wave forms) and to relate the function $?(x)$ to certain modular objects. In this talk, the Minkowski question mark-world is naturally integrated within a certain uniform construction with the classical world of modular forms for the full modular group. This construction leads to the new notion of mean-modular forms. The latter linear space with index $2k$ turns out to be canonically isomorphic to the space of quasi-modular forms of weight $2k$, weight $2k$ subspace of the graded ring generated by E_2, E_4 and E_6. Another modular connection is established by relating the Stieltjes transform of $?(x)$ with the Dedekind η-function.

Attainable numbers and the Lagrange spectrum

Dmitriy Gayfulin
(Moscow State University, Russia)

An irrational number α is called attainable if the inequality

$$\left|\alpha - \frac{p}{q}\right| \leq \frac{1}{\mu(\alpha)q^2}$$

holds for infinitely many integers p and q, where $\mu(\alpha)$ is the Lagrange constant defined as follows

$$\mu^{-1}(\alpha) = \liminf_{p\in\mathbb{Z},q\in\mathbb{N}} |q(q\alpha - p)|.$$

In my talk I will discuss the following question: does for any λ in the Lagrange spectrum exist some attainable irrational number α such that $\mu(\alpha) = \lambda$.

Quasi-modular functions and transcendental numbers

Yuri Nesterenko
(Moscow State University, Russia)

For the first time modular functions for the proof of transcendence of numbers were used in 1935 in joint article of K. Mahler and Y. Popken. They proved that at any complex number τ, $\Im\tau > 0$, the set $E_2(\tau), E_4(\tau), E_6(\tau)$ (Eisenstein series) contains at least one transcendental number. The first transcendence result about values of the modular invariant $j(\tau)$ has been proved in 1937 by Th. Schneider. For the proof he used properties of Weierstrass's elliptic functions, but this way seemed to him unnatural and Schneider formulated a problem to find the modular proof of his theorem. Then Mahler formulated a hypothesis about transcendence at any $\tau, \Im\tau > 0$, at least one of two numbers $e^{2\pi i\tau}$ and $j(\tau)$. Now a modular proof of Schneider's theorem is still not found. It is also open the complete hypothesis of Mahler-Manin about values of modular invariant and exponential function a^τ for algebraic $a \neq 0, 1$. In the talk we will discuss some results in this area and some attempts to use others modular and quasimodular functions, about further advances in this area.

On rational approximations to two irrational numbers

Artūras Dubickas
(Vilnius University, Lithuania)

Let α and β be two irrational real numbers satisfying $\alpha\pm\beta \notin \mathbb{Z}$. We prove several inequalities between $\min_{k\in\{1,\dots,n\}} \|k\alpha\|$ and $\min_{k\in S} \|k\beta\|$, where S is a set of positive integers, e.g., $S = \{n\}$, $S = \{1, \dots, n-1\}$ or $S = \{1, \dots, n\}$ and $\|x\|$ stands for the distance between $x \in \mathbb{R}$ and the nearest integer. We also give some constructions of α and β which show that the result of Kan and Moshchevitin (asserting that the difference between $\min_{k\in\{1,\dots,n\}} \|k\alpha\|$ and $\min_{k\in\{1,\dots,n\}} \|k\beta\|$ changes its sign infinitely often) and its variations are best possible. Some of the results are given in terms of the sequence $d(n) = d_{\alpha,\beta}(n)$ defined as the difference between reciprocals of these two quantities. In particular, we prove that the sequence $d(n)$ is unbounded for any irrational α, β satisfying $\alpha \pm \beta \notin \mathbb{Z}$.

Klein polyhedra as multidimensional analogue of continued fractions

Oleg German
(Moscow State University, Russia)

In our talk we shall give an overview of what is known by now about Klein polyhedra, one of the most natural multidimensional generalisations of continued fractions. This beautiful construction was proposed by F.Klein, but first nontrivial multidimensional results started appearing many decades after. And still, there are much more open questions than known answers.