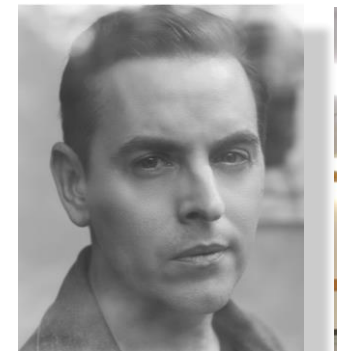


# Dynamic Competition Networks: detecting alliances and leaders

## Code & Data

[http://www.math.ryerson.ca/~abonato/papers/SurvivorBB\\_Data\\_BEGM](http://www.math.ryerson.ca/~abonato/papers/SurvivorBB_Data_BEGM)



Anthony Bonato (Ryerson)  
Rehan Malik (Ryerson)  
David Gleich (Purdue)

**Nicole Eikmeier**  
**Purdue University**

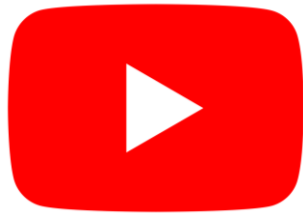
# In Traditional Networks, edges are “POSITIVE”



Edges are friendship



Edges are professional connections

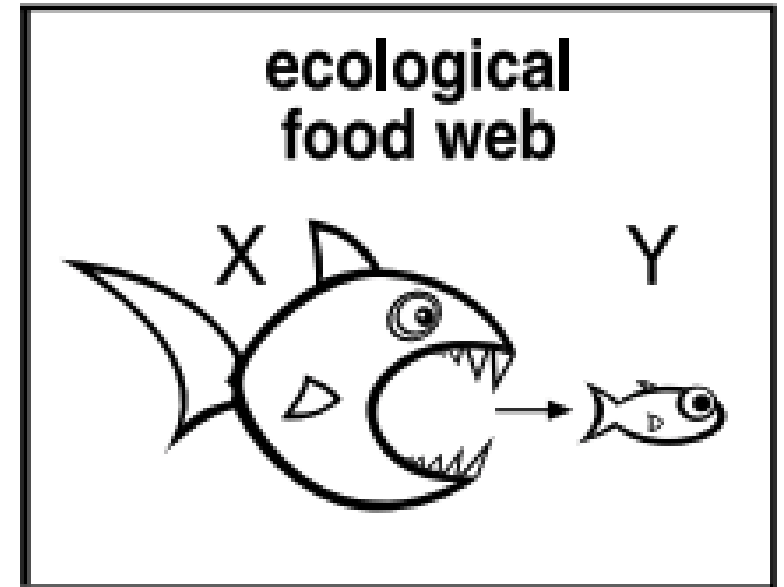


Edges are subscribers

# What about “NEGATIVE” interactions?

Examples:

- Geo-political Networks
- Competitive Social Games
- Food Webs
- Signed Networks (the negative portion)



Milo et. Al, Science, 2002

# Motivating Questions in Negative Interactions

Can we predict winners in the case of a competitive situation?

How do negative interactions drive link formation?

# Structural Balance Theory

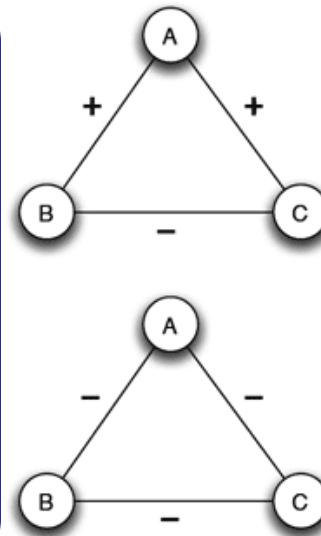
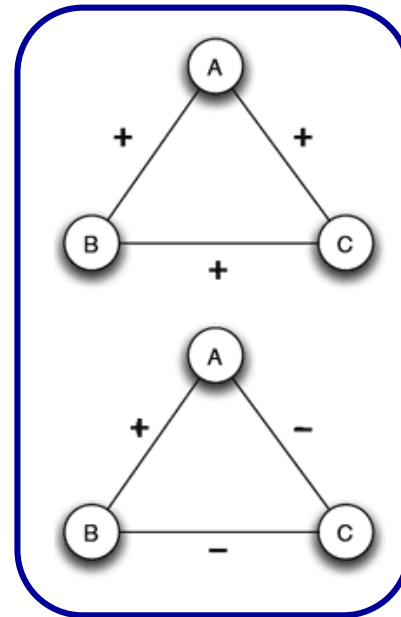
Heider, 1958

Easley, Kleinberg, 2010

Considers triads of nodes

Triads seek *closure*

balanced



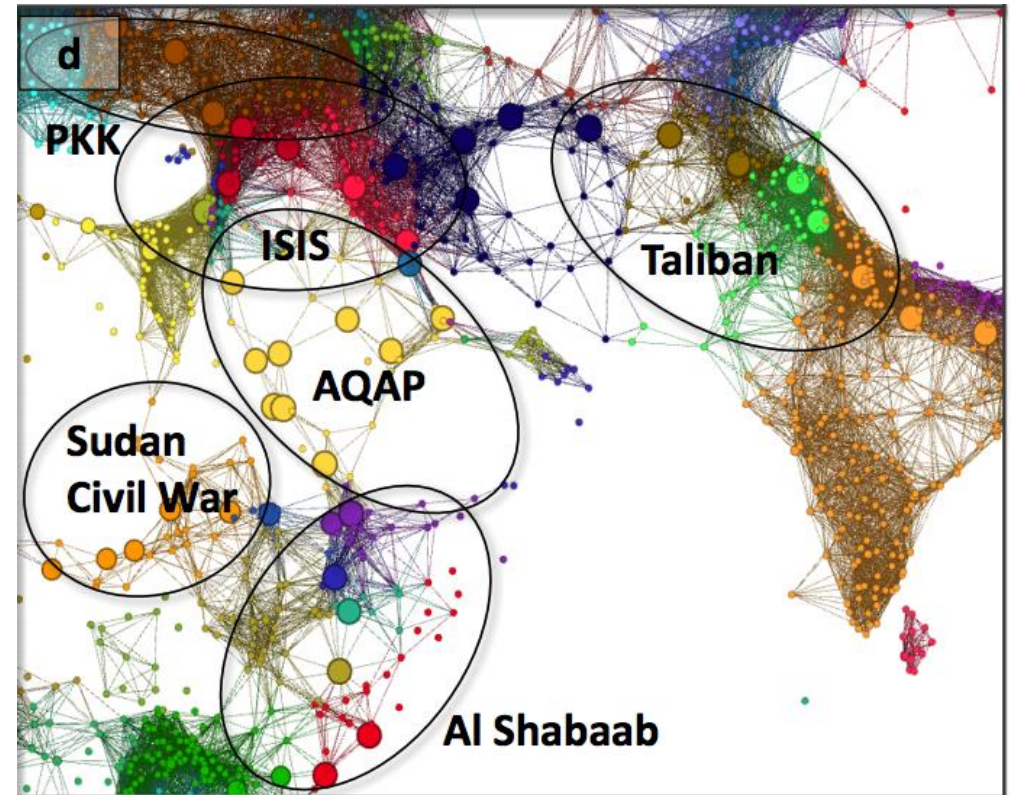
unbalanced

# Related Work – Negative Interactions

Guo et. al, Preprint, 2018

Form a network based on spatial location of cities and predict the rise of conflict and violence

Their analysis is based on degree and betweenness



# Related Work – Negative Interactions

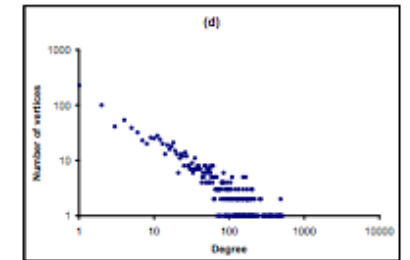
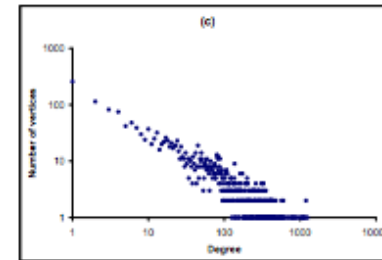
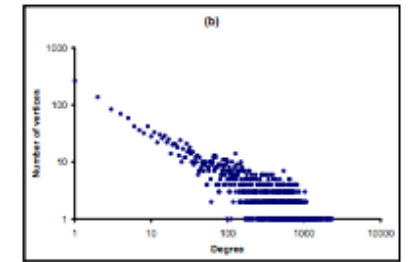
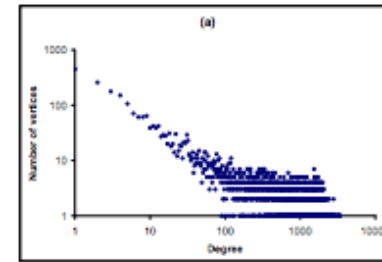
Boginski, Butenko, Pardalos, 2003

Study negative interactions between stocks on the stock market

Analysis on the graph:

Power-law distribution in the degrees

Find independent sets, sets of stocks whose prices change similarly over time



# Dynamic Competition Networks

**Nodes:** people/organizations

**Edges:** directed edge  $(u,v)$  exists if  $v$  is a competitor of  $u$

Edges are added over time



# Social Game Shows





**Nodes:** players

**Edges:**  $(u,v)$  exists  
if player  $u$  votes  
against player  $v$

There can be multiple  
edges between two  
nodes



# Network Metrics

**In-Degree**: The number of edges coming into a node

Common out-neighbor (**CON**) scores count the total number of times a node has a similar out-neighbor as other nodes

$$CON(u, v) = |\{z : z \text{ is a common out neighbor of } u, v\}|$$

$$CON(u) = \sum_{v \in V(G)} CON(u, v)$$

The **closeness** of a node is  $(\sum_y d(x, y))^{-1}$

A set of nodes is **near-independent** if its edge density is small

$$ED(S) = \frac{|E(S)|}{\binom{|S|}{2}}$$

## **alliances:**

clusters of nodes who pool influence to achieve objectives

## **leaders:**

high standing in the network

edges from leaders may influence edge creation among other nodes

# Dynamic Competition Hypothesis (DCH)

Bonato, Eikmeier, Gleich, Malik, 2018

Dynamic Competition Networks satisfy the following rules:

- Alliances are near independent sets

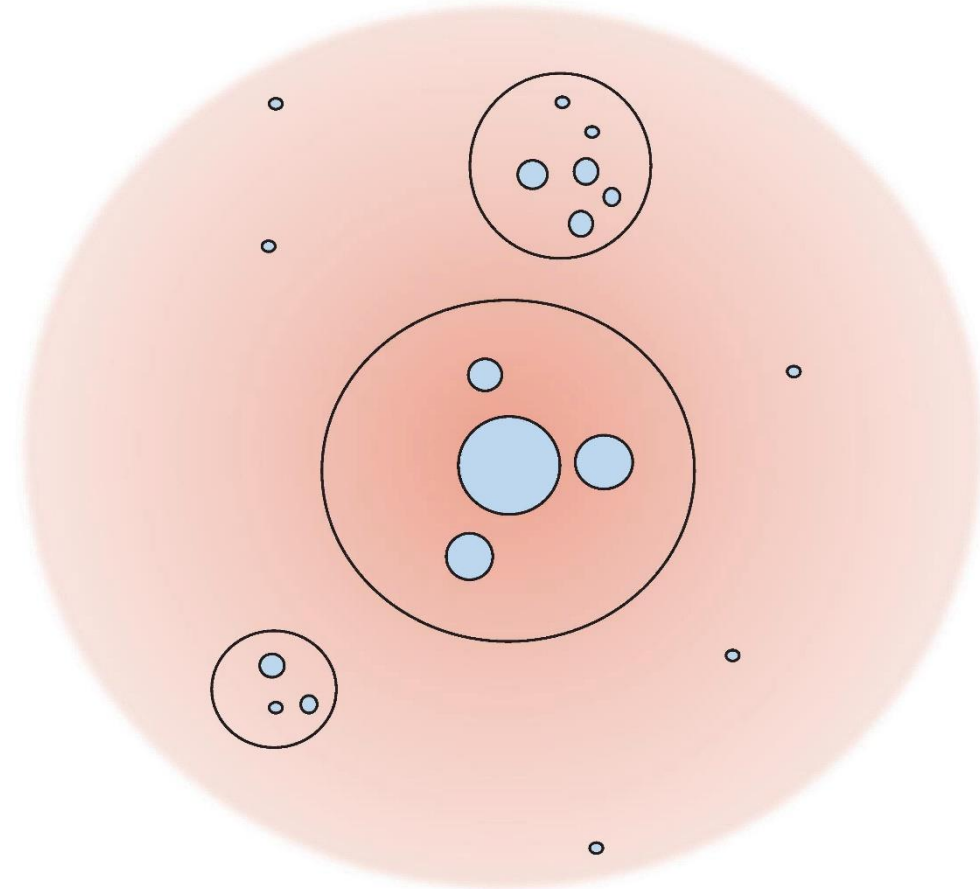
- Strong alliances have low edge density

- Members of an alliance with high CON score are more likely leaders

- Leaders exhibit high closeness, high CON scores, and low in-degree

While we expect leaders to be in an alliance, leaders are determined via global metrics

# DCH Visualized





# Social Game Shows





## Survivor

35 US seasons

[http://survivor.wikia.com/wiki/  
Main\\_Page](http://survivor.wikia.com/wiki/Main_Page)



## Big Brother

20 US seasons

[http://bigbrother.wikia.com/wiki/  
Big\\_Brother\\_Wiki](http://bigbrother.wikia.com/wiki/Big_Brother_Wiki)





# Survivor: Game Changers

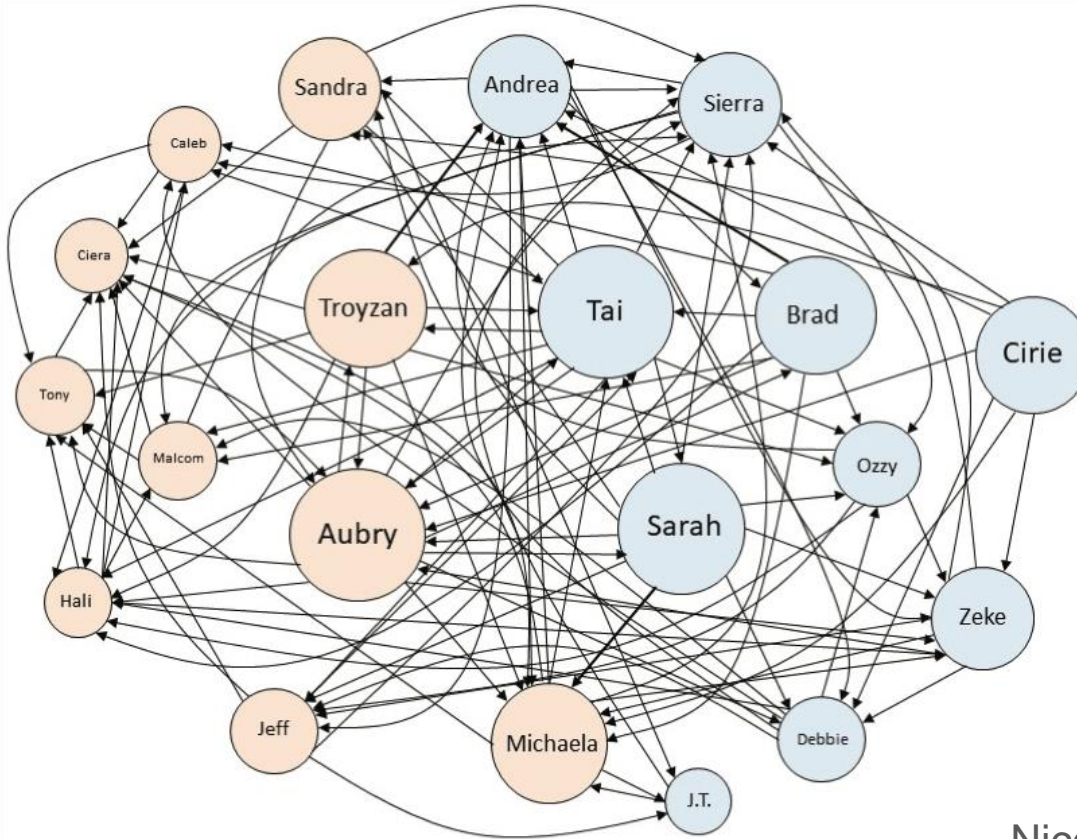
DCH:

Alliances are near independent sets

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Members of an alliance with high CON score are more likely leaders

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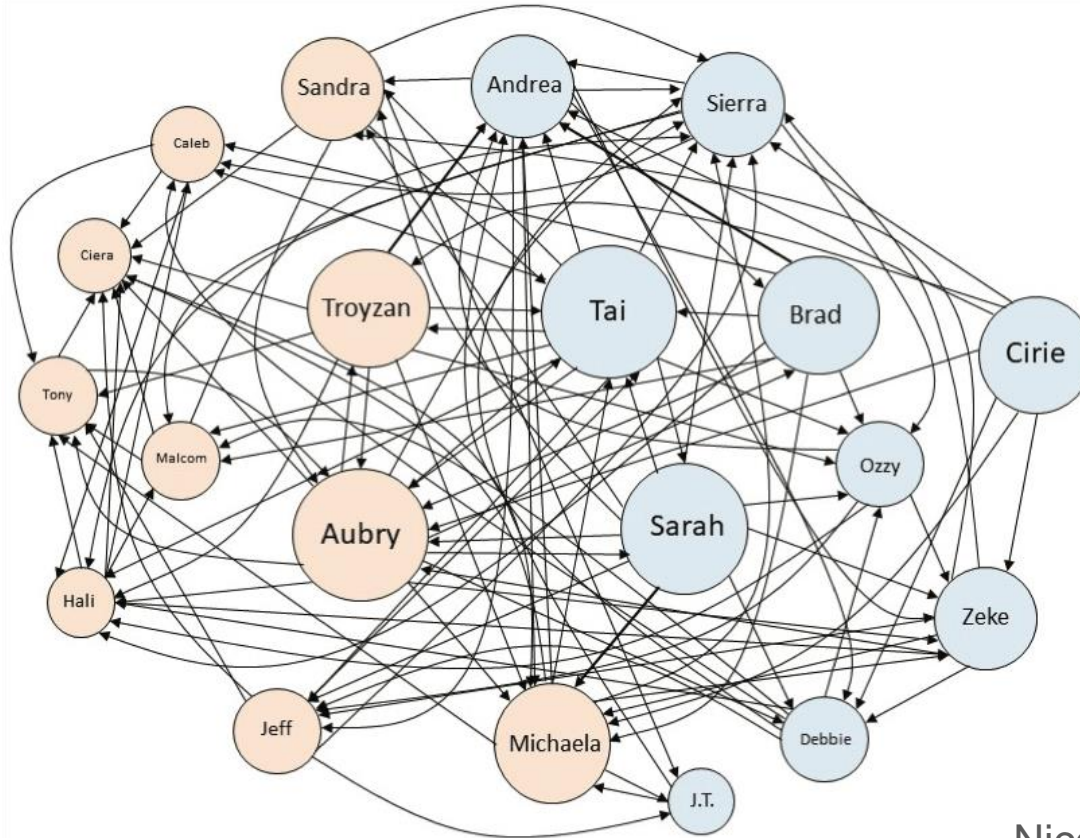
Game Changers				
Name	ID	OD	C	CON
Sarah	3	13	0.692	64
Brad	2	12	0.643	49
Troyzan	2	12	0.643	55
Tai	12	13	0.72	56
Aubry	9	13	0.72	61
Cirie	0	8	0.613	45
Michaela	11	11	0.643	51
Andrea	14	8	0.581	39
Sierra	15	7	0.581	34
Zeke	11	6	0.6	39
Debbie	6	7	0.545	32
Ozzy	7	4	0.5	22
Hali	8	5	0.474	28
Jeff	6	5	0.529	33
Sandra	5	5	0.581	34
J.T.	3	2	0.45	18
Malcolm	5	3	0.439	24
Caleb	5	3	0.4	21
Tony	7	2	0.439	15
Ciera	9	1	0.4	8

# Survivor: Game Changers

Edge Density: 0.233

DCH:

Alliances are near independent sets  
 Strong alliances have low edge density  
 Members of an alliance with high CON score are more likely leaders  
 Leaders exhibit high closeness, high CON scores, and low in-degree



Game Changers				
Name	ID	OD	C	CON
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Brad	2	12	0.643	49
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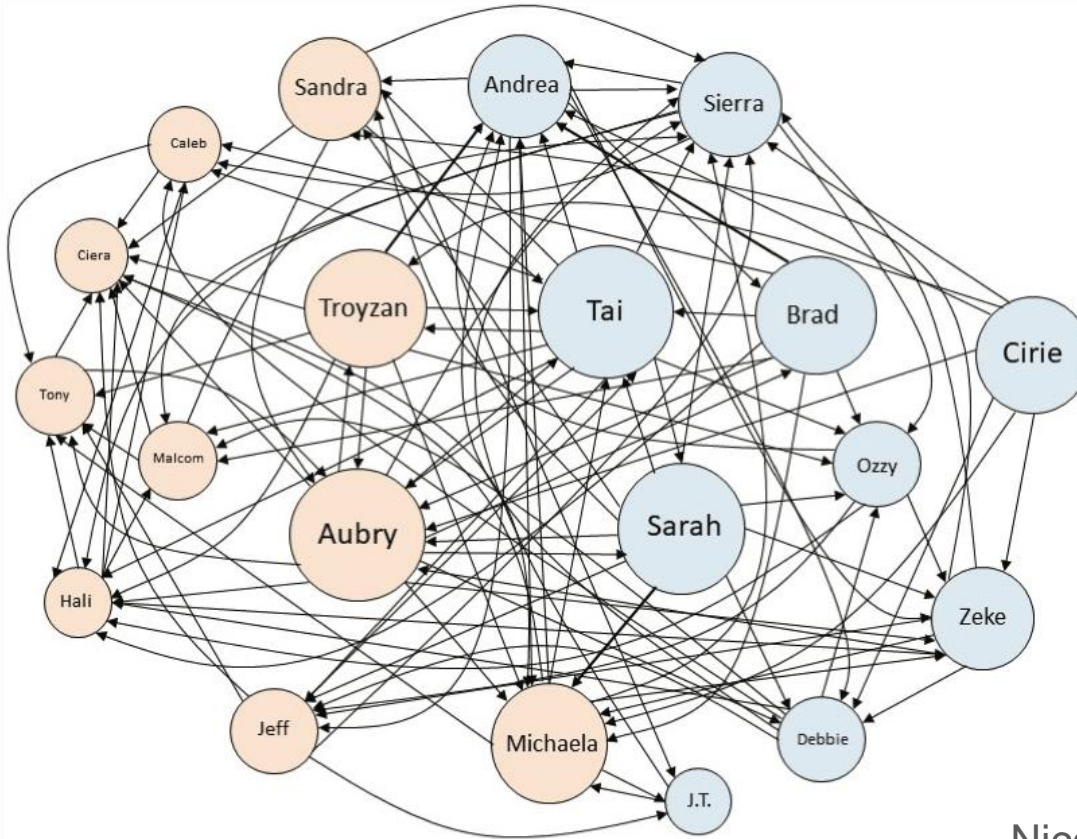


# Survivor: Game Changers

Edge Density: 0.066

DCH:

Alliances are near independent sets  
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Game Changers				
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# CON is a good choice of metric

DCH:

Alliances are near independent sets

Strong alliances have low edge density

Members of an alliance with high CON score are more likely leaders

Leaders exhibit high closeness, high CON scores, and low in-degree

		CON
<b>Survivor</b>	Top 3	68.6
	Top 5	94.3
<b>Big Brother</b>	Top 3	60.0
	Top 5	70.0

★ there are only 20 seasons of Big Brother, with sometimes as few as 10 players

# Edge Density of Alliances

DCH:

Alliances are near independent sets  
Strong alliances have low edge density

Members of an alliance with high CON score are more likely leaders  
Leaders exhibit high closeness, high CON scores, and low in-degree

To what extent do alliances have a smaller edge density than the edge density of the entire graph?

60% of Survivor seasons

95% of Big Brother Seasons

Alliances are complicated to analyze

# Thank you!



Anthony Bonato (Ryerson)  
Rehan Malik (Ryerson)  
David Gleich (Purdue)

## Code & Data

[http://www.math.ryerson.ca/~abonato/papers/SurvivorBB\\_Data\\_BEGM](http://www.math.ryerson.ca/~abonato/papers/SurvivorBB_Data_BEGM)

## Contributions

Notion of a Dynamic Competition Network

Dynamic Competition Hypothesis:

- detects leaders

- measures strength of alliances

Test of DCH on social game shows Survivor and Big Brother

## Questions & Future Work

Does the DCH apply to other types of networks?

Can we invert the DCH to determine low-ranked members?

Apply the DCH to international seasons of Survivor and Big Brother

Analyze Social Network Games at earlier stages in formation of the network