Decision tree analysis with SilverDecisions.pl

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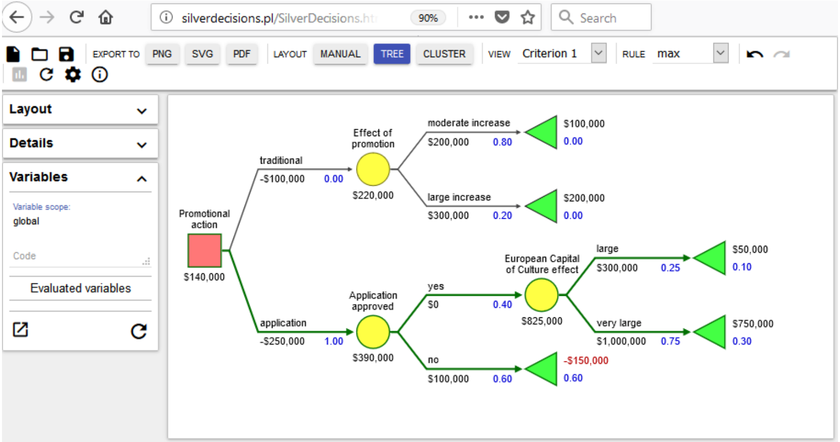
SilverDecisions.pl user interface



Decision tree

A decision tree DT is a tuple (G, y, p)

- Directed graph $G = (V, E)$, $E \subset V^2$, with set of nodes V split into three disjoint sets $V = \mathcal{D} \cup \mathcal{C} \cup \mathcal{T}$
  - $\mathcal{D}$ decision node
  - $\mathcal{C}$ chance node
  - $\mathcal{T}$ terminal node
  - for each edge $e \in E$
    - $e_1 \in V$ denote its first element (parent node)
    - $e_2 \in V$ denote its second element (child node)
- payoff function: $y \colon E \to \mathbb{R}$
- probabilities function: $p \colon \{e \in E : e_1 \in \mathcal{C}\} \to [0, 1]$
- additional conditions: has root node, terminal nodes terminal nodes $\mathcal{T}$ have no children, $p(\cdot)$ is properly defined (i.e. probabilities at chance node sum up to 1)

Decision tree

Goal: find a decision function
d, $d\colon \{e \in E : e_1 \in \mathcal{D}\} \to \{0,1\}$,
$\forall v \in \mathcal{D} : \sum_{e \in E, e_1 = v} d(e) = 1$

that maximizes the payoff

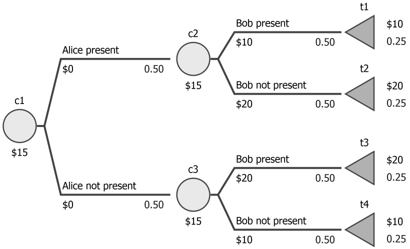
$$P(DT,d) = \begin{cases} 0 & \text{if} \quad r(DT) \in \mathcal{T} \\ \sum_{e:e_1=r(DT)} p(e)\left(y(e) + P\left(DT(e_2), d\right)\right) & \text{if} \quad r(DT) \in \mathcal{C} \\ \sum_{e:e_1=r(DT)} d(e)\left(y(e) + P\left(DT(e_2), d\right)\right) & \text{if} \quad r(DT) \in \mathcal{D} \end{cases}$$

where $r(DT)$ denotes the root of DT.

A strategy d maximizing $P(DT,d)$ is called *P-optimal*

Decision tree types

- Separable (changing the probabilities in one chance node does not automatically require changing any probabilities in any other chance node)
- Non-Separable (see the picture below)



Sensitivity analysis

1. $s \colon \mathcal{C} \to [0, 1]$
   function representing whether a given node should be subject to sensitivity analysis. $s(c) = 1$ denotes that c should be subject to sensitivity analysis
   $s(c) = 0$ denotes that probabilities are given precisely
   Values between 0 and 1 could denote various degrees of ambiguity
2. Stability analysis
   how stable a P-optimal strategy is
3. Perturbation analysis
   optimal strategy for various scenarios of probability perturbation

Sensitivity analysis - stability

- distance between two decision trees $DT' = ((V,E),p',y')$ and $DT = ((V,E),p,y)$

$$||DT, DT'||_s = \max_{e \in E, e_1 \in \mathcal{C}} \frac{|p'(e) - p(e)|}{s(e_1)}.$$

- The P-optimal strategy d is ε-stable, if it is also P-optimal for any DT' such that $||DT, DT'||_s \leq \varepsilon$
- stability index of a P-optimal strategy d:

$$I(DT, d, s) = \sup\{\varepsilon \in [0, +\infty] \colon d \text{ is } \varepsilon\text{-stable}\}$$

Theorem

Take a separable proper decision tree, DT, with a sensitivity function, $s(\cdot)$.

For any two P-optimal strategies, d_1 and d_2, we have $I(DT, d_1, s) = I(DT, d_2, s)$.

Sensitivity analysis - perturbation

- minimum positive sensitivity value
  $\widetilde{s} = \min\{s(v) : v \in \mathcal{C}, s(v) > 0\}$
- worst-case-tending expected payoff:

$$P_{\min}(DT, d, s, \varepsilon) = \min_{\{DT' : \, ||DT, DT'||_s \leq \varepsilon\}} P(DT', d).$$

- best-case-tending perturbation:

$$P_{\max}(DT, d, s, \varepsilon) = \max_{\{DT' : \, ||DT, DT'||_s \leq \varepsilon\}} P(DT', d)$$

Sensitivity analysis - perturbation cont.

- divergence function

$$D_{KL}(P||Q) = \sum_{i \in A} P(i) \log\left(\frac{P(i)}{Q(i)}\right),$$

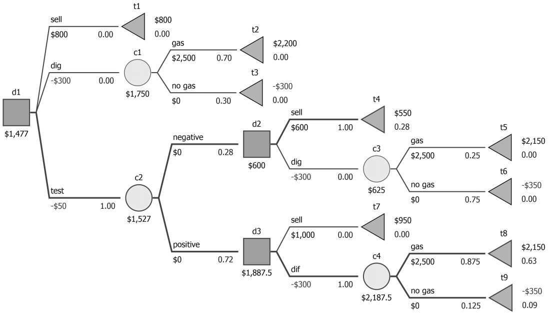
$$\begin{aligned} &\text{minimize} && D_{KL}(\mathbf{x}||\mathbf{p}) \\ &\text{subject to:} && D_{KL}(\mathbf{x}||\mathbf{u}) = \theta \wedge \mathbf{x} \geq (0) \wedge \sum_{i=1}^{n} x_i = 1, \end{aligned}$$

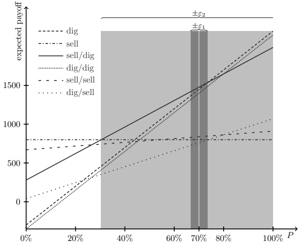
where $\mathbf{u} = (\frac{1}{n}, \ldots, \frac{1}{n})$ (of length n)

- The solution for the above optimization problem for various values of θ yields $\mathbf{x}$ changing along the path given by the following soft-max formula with parameter $\gamma \in [0, +\infty[$:

$$x_i = \frac{p_i^\gamma}{\sum_{j=1}^{n} p_j^\gamma}.$$

Non separable decision tree - sample





$$P(\text{pos. test}) = \text{sensitivity} \times P(\text{gas}) + (1 - \text{specificity}) \times P(\text{no gas}),$$

$$P(\text{neg. test}) = (1 - \text{sensitivity}) \times P(\text{gas}) + \text{specificity} \times P(\text{no gas}),$$

$$P(\text{gas}|\text{pos. test}) = \frac{\text{sensitivity} \times P(\text{gas})}{P(\text{pos. test})},$$

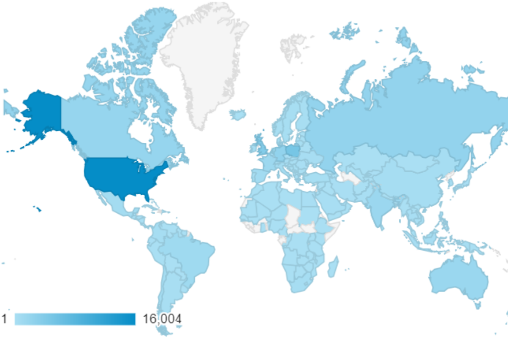
$$P(\text{gas}|\text{neg. test}) = \frac{(1 - \text{sensitivity}) \times P(\text{gas})}{P(\text{neg. test})},$$

Software

- SilverDecisions.pl (`http://silverdecisions.pl/`)
    - Graphical user interface, web based
    - Over 50'000 unique users (2'200 from Russia)
    - Javascript
    - Exports files to JSON
    - Fully open source - hosted on Git
- Chondro (`https://github.com/pszufe/chondro`)
    - A callable API
    - Python
    - Imports JSON decision tree definitions
        - Internal format (a nested Python dictionary representing a tree structure)
        - Trees exported from SilverDecision
    - Fully open source - hosted on Git

SilverDecisions.pl Users

	Country	Users	Sessions
		52,854 % of Total: 100.00% (52,854)	77,293 % of Total: 100.00% (77,293)
1.	United States	**16,004** (30.33%)	21,710 (28.09%)
2.	Poland	**4,969** (9.42%)	11,349 (14.68%)
3.	United Kingdom	**3,754** (7.12%)	4,723 (6.11%)
4.	Germany	**2,359** (4.47%)	3,175 (4.11%)
5.	Russia	**2,280** (4.32%)	3,077 (3.98%)
6.	Canada	**1,966** (3.73%)	2,515 (3.25%)
7.	Netherlands	**1,657** (3.14%)	2,466 (3.19%)
8.	Australia	**1,557** (2.95%)	2,289 (2.96%)
9.	India	**1,493** (2.83%)	1,896 (2.45%)
10.	France	**1,294** (2.45%)	1,630 (2.11%)



Chondro - typical usage scenario

1. Create a JSON representation of a DT (e.g. by saving a DT from SilverDecisions
2. Load the tree and use the `solve_tree` function to calculate optimal decision for the DT.
3. Perform the sensitivity analysis
   - use `find_stability` to calculate the stability
   - use `find_perturbation_mode` to calculate $P_{mode,\varepsilon}$
   - use `find_perturbation_pessopty` to calculate $P_{\min,\varepsilon}$ and $P_{\max,\varepsilon}$

```
d1:decision
  *c1:chance (ev=48)
    t1:p=3/5 final [60]
    t2:p=2/5 final [30]
   c2:chance (ev=44)
    t3:p=7/10 final [20]
    t4:p=3/10 final [100]
   c3:chance (ev=44)
    t5:p=4/5 final [40]
    t6:p=1/5 final [60]
```

Summary - results

1. Conceptual framework for sensitivity analysis of decision trees;
2. Methodology for performing SA when values in several nodes change simultaneously
3. Open Source software implementation that enables practical application of the concepts discussed in the paper
    - SilverDecisions.pl
        - currently over 50'000 users (most from the USA)
        - 2'200 users from Russia
    - Chondro (online Python library for tree stability analysis)

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