

Stability of intersections of graphs in the plane and the van Kampen obstruction

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Abstract

Abstract. A map $\varphi : K \rightarrow \mathbb{R}^2$ of a graph K is **approximable by embeddings**, if for each $\varepsilon > 0$ there is an ε -close to φ embedding $f : K \rightarrow \mathbb{R}^2$. Analogous notions were studied under the names of **cluster planarity** and **weak simplicity**. We present criteria for approximability by embeddings (P. Minc, 1997, M. Skopenkov, 2003) and their algorithmic corollaries.

A map $\varphi : K \sqcup L \rightarrow \mathbb{R}^2$ of the disjoint union of graphs K and L is **approximable by maps with disjoint images**, if for each $\varepsilon > 0$ there is an ε -close to φ map $f : K \sqcup L \rightarrow \mathbb{R}^2$ such that $f(K) \cap f(L) = \emptyset$. We present open problems on this notion.

We introduce **the van Kampen (or Hanani-Tutte) obstructions** for these properties, as well as their generalizations to higher dimensions and to r -tuple intersections. We present the completeness result of this obstruction (D. Repovš and A. Skopenkov, 1998) and its algorithmic corollary.