Комбинаторика слов /Combinatorics on Words

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Аннотация

Комбинаторика слов-это раздел математики и теоретической информатики, который применяет комбинаторный анализ к конечным или бесконечным словам. Эта отрасль развивалась из нескольких отраслей математики: теории чисел, теории групп, вероятностей и, конечно же, комбинаторики. Он имеет связи с различными компьютерными темами, такими как текстовые алгоритмы, поиск шаблонов и сжатие текста.

Course Description

Word combinatorics is a branch of mathematics and theoretical computer science that applies combinatorial analysis to finite or infinite words. This branch has developed from several branches of mathematics: number theory, group theory, probabilities and of course combinatorics. It has links with various computer topics, such as text algorithms, pattern search and text compression.

Course Outline

- **Introduction:** What is Combinatorics on words?, Definition of word, Some motivation examples, Basic examples in: Number theory, Graph theory, Symbolic dynamics, Discrete geometry, Group theory, Bio-informatics, Introduction to morphic words

- **General notions:** Languages, Affixes, Distance of words, *p*-adic valuation, *p*-adic absolute, Open balls, Converges, Periodically, Compactness, Concatenation, Semigroup and monoid, Morphism, Prolongable, Complexity function, Entropy, De Bruijn graph, Circular word, Rauzy graph;

- **Periodicity in words:** Powers and periods, Primitive, Fine–Wilf's theorem, Lyndon word, Equations on words, Bordered and unbordered words, Code, aperiodic necklace; Some theorems about Lyndon word;

- **Sturmain words:** Finite and infinite Sturmain word, Fibonacci word, Mechanical word, Standard word, Continuous fraction, Billiard words, Beatty sequences, Coding of irrational rotation, Non-binary words, Associated real numbers;

- **Automatic sequences:** *k*-uniform morphisms, Factor complexity, Adamczewski–Bugeaud's theorem, Cobham's theorem, Frequencies, The Fibonacci word (or any Sturmian word) is not *k*-automatic, Thue–Morse word, Burnside's problem, Logical characterization, Expansion.

Prerequisites/Corequisites

Mathematical Analysis, Discrete Analysis, Linear Algebra, Group Theory, Number Theory

Main References

1. V. Berthe, M. Rigo (Ed.), Combinatorics, Automata and Number Theory, Cambridge Univ. Press (2010).
2. G. Rozenberg, A. Salomaa (Ed.), Handbook of Formal Languages, Springer (1997).
3. D. Lind, B. Marcus, An Introduction to Symbolic Dynamics and Coding, Cambridge Univ. Press (1995).
4. M. Lothaire, Combinatorics on Words, Cambridge University Press, Cambridge, 1997.
5. M. Lothaire, Algebraic Combinatorics on Words, Cambridge University Press, Cambridge, 2002.
6. J. Karhumaki, Combinatorics of words, Lecture Notes, Univ.of Turku.
7. M. Rigo, Formal Languages, Automata and Numeration Systems 1, Wiley-ISTE, 2014.

Grading Policy

The grade will count the assessments using the following proportions:

- **50%** of your grade will be determined by 5 series hometasks (10% each).
- **20%** of your grade will be determined by 2 projects (10% each).
- **30%** of your grade will be determined by 1 oral exam.

Hometasks:

A series of exercises is provided for each section of General notions, Periodicity in words, Sturmain words and Automatic sequences. The last exercise is general.

Projects:

You will have two projects: practical and theoretical.

In-person exam:

It has three parts: 1) definitions (5%), 2) theorems with proving (20%), and 3) problems (5%).

Online exam:

You have to prepare two slides for each project, record your presentations and upload them in YouTube.

Заведующий кафедрой
дискретной математики А.М. Райгородский