We study the following computational problem: for which values of k, the majority of n bits MAJ_n can be computed with a depth two formula whose each gate computes a majority function of at most k bits? The corresponding computational model is denoted by $\mathrm{MAJ}_k \circ \mathrm{MAJ}_k$. We observe that the minimum value of k for which there exists a $\mathrm{MAJ}_k \circ \mathrm{MAJ}_k$ circuit that has high correlation with the majority of n bits is equal to $\Theta(n^{1/2})$. We then show that for a randomized $\mathrm{MAJ}_k \circ \mathrm{MAJ}_k$ circuit computing the majority of n input bits with high probability for every input, the minimum value of k is equal to $n^{2/3+o(1)}$. We show a worst case lower bound: if a $\mathrm{MAJ}_k \circ \mathrm{MAJ}_k$ circuit computes the majority of n bits correctly on all inputs, then $k \geq n^{13/19+o(1)}$. This lower bound exceeds the optimal value for randomized circuits and thus is unreachable for pure randomized techniques. For depth 3 circuits we show that a circuit with $k = O(n^{2/3})$ can compute MAJ_n correctly on all inputs.

The talk is based on joint results with Alexander Kulikov.