

The domination number of the graph defined by two levels of the n -cube

Gyula O.H. Katona
MTA Rényi Institute, Budapest, Hungary

Consider all k -element subsets and ℓ -element subsets ($k > \ell$) of an n -element set as vertices of a bipartite graph. Two vertices are adjacent if the corresponding ℓ -element set is a subset of the corresponding k -element set. Let $G_{k,\ell}$ denote this graph. The domination number of $G_{k,1}$ is exactly determined. We also prove that $\gamma(G_{k,2})$ is asymptotically equal to

$$\frac{k+3}{2(k-1)(k+1)}n^2$$

for $k \geq 3$. The upper estimate is proved by a random construction. We also suggest a way to find a deterministic construction, but it is completed only for $k = 3$ and 4.

Joint work with Leila Badakhshian and Zsolt Tuza.