

Geometry on Words

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Class Hours:

Class Room: Online

Intended Audience

B.Sc students or M.Sc students in mathematics.

Course Description

Many interesting examples of words (sequences of letters) are connected with number theory and have explicit geometric interpretation, based on some "self-similarity".

For example, Christoffel words have connections with continued fractions. The Tribonacci word 121312112131... is a fixed point of the substitution $1 \rightarrow 12, 2 \rightarrow 13, 3 \rightarrow 1$ and encodes a fractal subdivision and a rotation of a 2-dimensional torus.

We will investigate these (and not only these) examples and will try to understand connections between combinatorics, number theory, geometry and dynamical systems.

The topics covered in the class will include (but are not limited to):

Course Outline

- **Rauzy Fractals:** 1) Substitution sequences, examples and basic properties 2) Pisot numbers 3) Definition of Rauzy fractals and examples of fractals 4) Hausdorff distance, contracting maps and equations on sets 5) Connection with rotation of torus, Pisot conjecture
- **Integer Geometry:** 1) Definition of a continued fraction and their properties 2) Geometric continued fractions and their properties 3) Integer angle, triangle and trigonometry and Pick's Formula 4) Gauss's reduction theory and linear algebraic methods
- **Christoffel words:** 1) Combinatorics on Christoffel words 2) Geometric aspects of Christoffel words 3) Continued fractions and Christoffel words 4) Markov numbers and their properties 5) Farey sequence
- **Outer Billiard:**

Research Talks

- **Alexei Belov:** Words obtained from values of polynomials in integer points
- **Alexei Semenov:** TBA

Prerequisites/Corequisites

All necessary prerequisites are covered.

Main References

TBA

Grading Policy

The grade will count the assessments using the following proportions:

- 40% of your grade will be determined by 4 series hometasks (10% each).
- 60% of your grade will be determined by 1 projects.

Hometasks:

Exercises consist of solved problems and research problems based on the ability of the students in the class.